



Swiss Doctoral Program in Mathematics

Cours du semestre d'automne 2009 à l'EPFL

- Title** Numerical ranges of matrices and linear operators
- Instructor** Christiane Tretter (Bern)
- Time** September 16 - December 16, 2009, Wednesday, 15:45 - 17:00
- Place** EPF Lausanne, room MA A1 12
- Audience** PhD students, advanced Master students
- Content** The course will focus on various concepts of numerical ranges for matrices and linear operators in Hilbert spaces, including the classical numerical range $W(A) = \{(Ax, x) : \|x\| = 1\} \subset \mathbb{C}$, the more recent quadratic (or more generally) block numerical range for partitioned matrices and block operator matrices, and numerical ranges of matrix functions or operator functions. All these numerical ranges have a number of useful properties which will be addressed in the lecture course: they yield e.g. enclosures for eigenvalues and spectrum and provide estimates of the resolvent. This is particularly important when dealing with non-symmetric matrices or operators. In the symmetric case, the functional defining the classical numerical range can be used to establish variational principles and estimates for eigenvalues. More advanced results are obtained with the quadratic numerical range; it provides a criterion for non-symmetric 2×2 block (operator) matrices to be block diagonalized and, as a consequence, for the existence of solutions of algebraic Riccati equations. Moreover, there are still some open problems waiting to be solved: for example, the question whether one of the (at most) two components of the quadratic numerical range may have a hole in the complex plane.
- Evaluation** On request
- Prerequisite** It is possible to benefit from the lecture even if one is only interested in the matrix case. In fact, many of the results are new even in finite dimensions; in particular, the open problem is still unsolved in the matrix case. Therefore linear algebra and some basics about Hilbert spaces and complex analysis are sufficient. For people who, in addition, want to understand the linear operator case, a basic course in functional analysis suffices.
- Keywords** Numerical ranges, spectrum, matrices, linear operators
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