Stochastic-Lagrangian Modeling of Multiphase Flow in Porous Media

Manav Tyagi

Patrick Jenny

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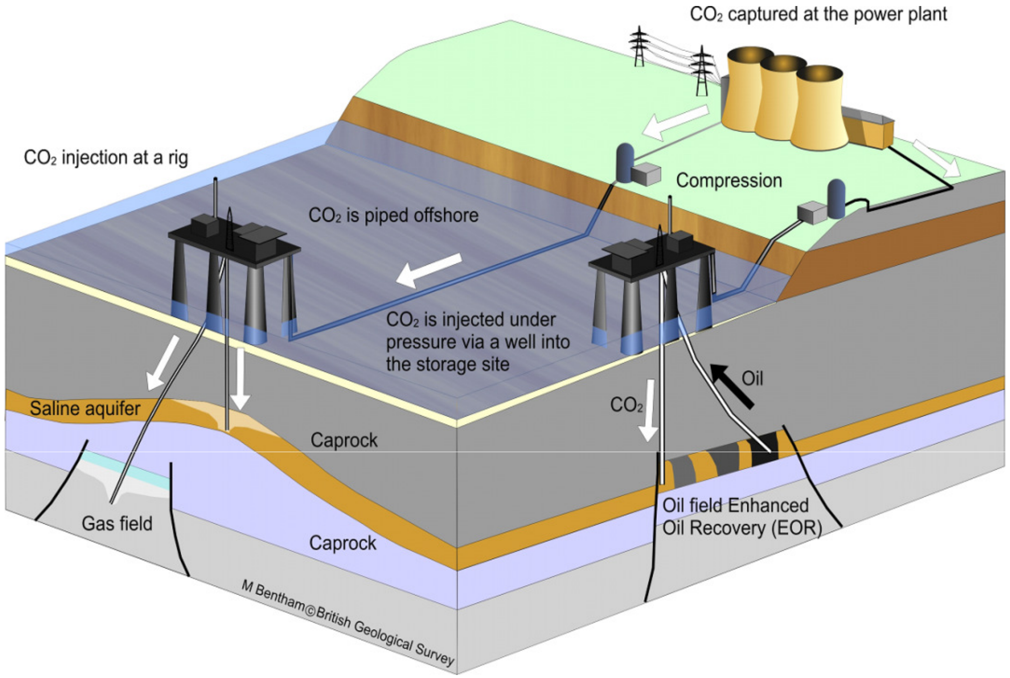
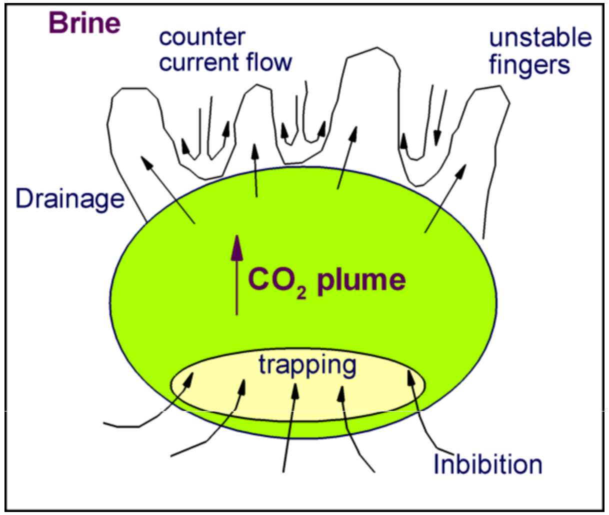
Institute of Fluid Dynamics, ETH Zurich, Switzerland

Outline

- Motivation
- Multiphase Flow in Porous Media
- Stochastic Framework
- Modeling and Simulation Results
- Conclusion and Outlook

CO_2 Storage in Geological Sites

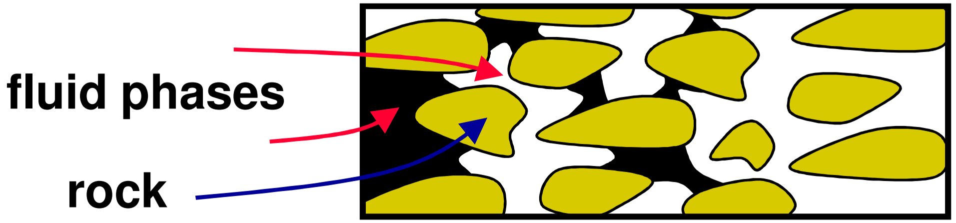




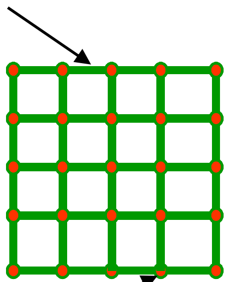
Complex processes

- Trapping (structural and residual)
- Dissolution of CO_2 into brine
- Reaction of acidic brine with rock

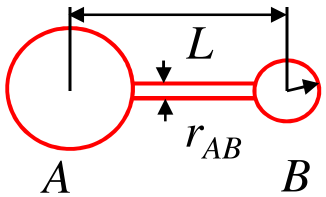
Modeling of flow at different scales



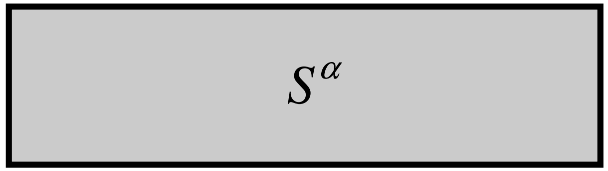
Small scale model



$$F_{AB} = \frac{\pi r_{AB}^4}{8L\mu_{AB}}\left(p_A - p_B\right)$$



Large scale model



Darcy's law $F^\alpha = -\lambda^\alpha\left(S^\alpha\right)\nabla p^\alpha$

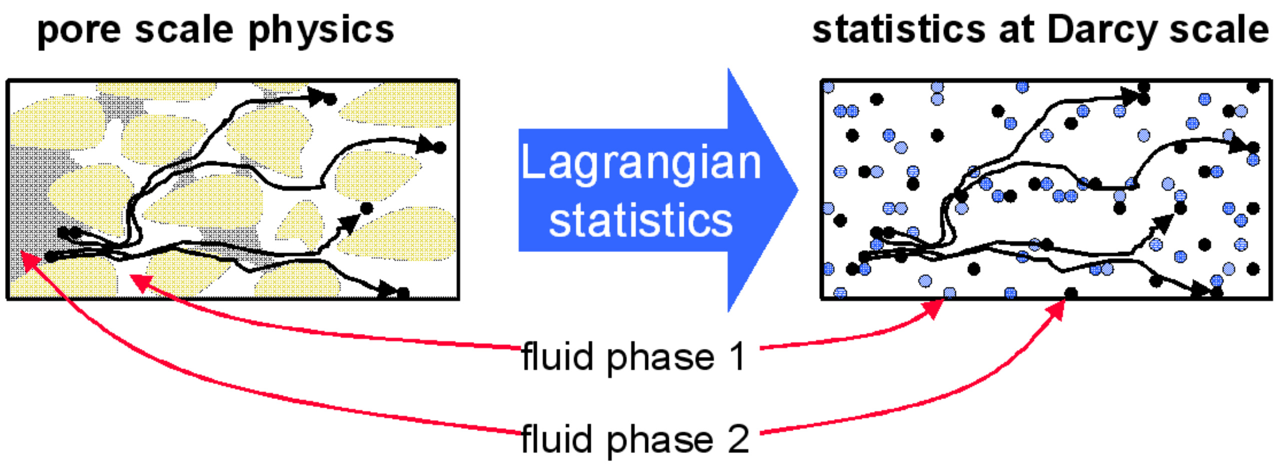
Transport $\frac{\partial S^\alpha}{\partial t} + \nabla \cdot F^\alpha = q^\alpha$

Flow $\nabla \cdot \left(\sum \lambda^\alpha \nabla p^\alpha\right) = -q$

…however

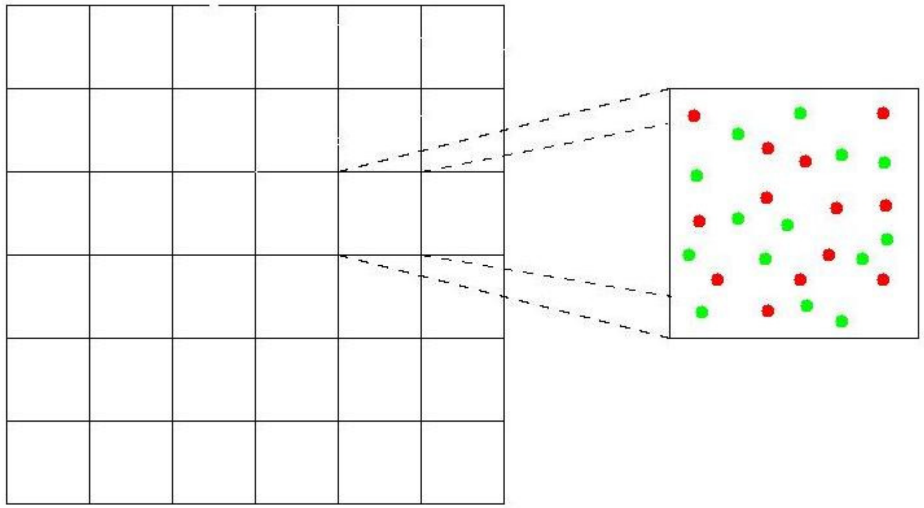
- Complex nonlinear small scale processes
- Simple up-scaling of pore scale flow is not enough
- Need of a framework for consistent up-scaling

Stochastic-Lagrangian Framework



Computational Framework

Particles represent phases
Green particles- gas phase
Red particles- liquid phase



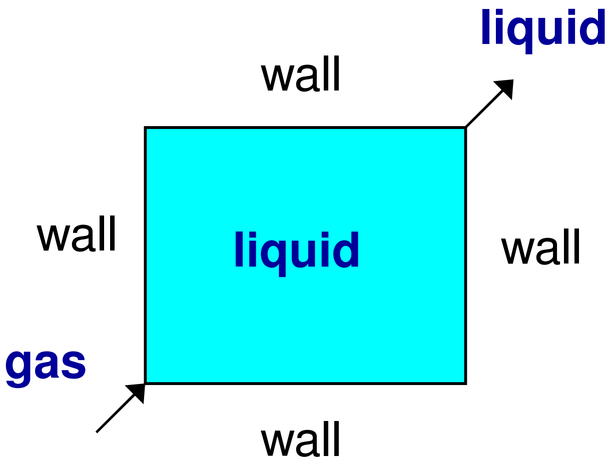
Cell averaged saturation

$$S^{\alpha} = \frac{\text{number of particles of phase } \alpha}{\text{total number of particles}}$$

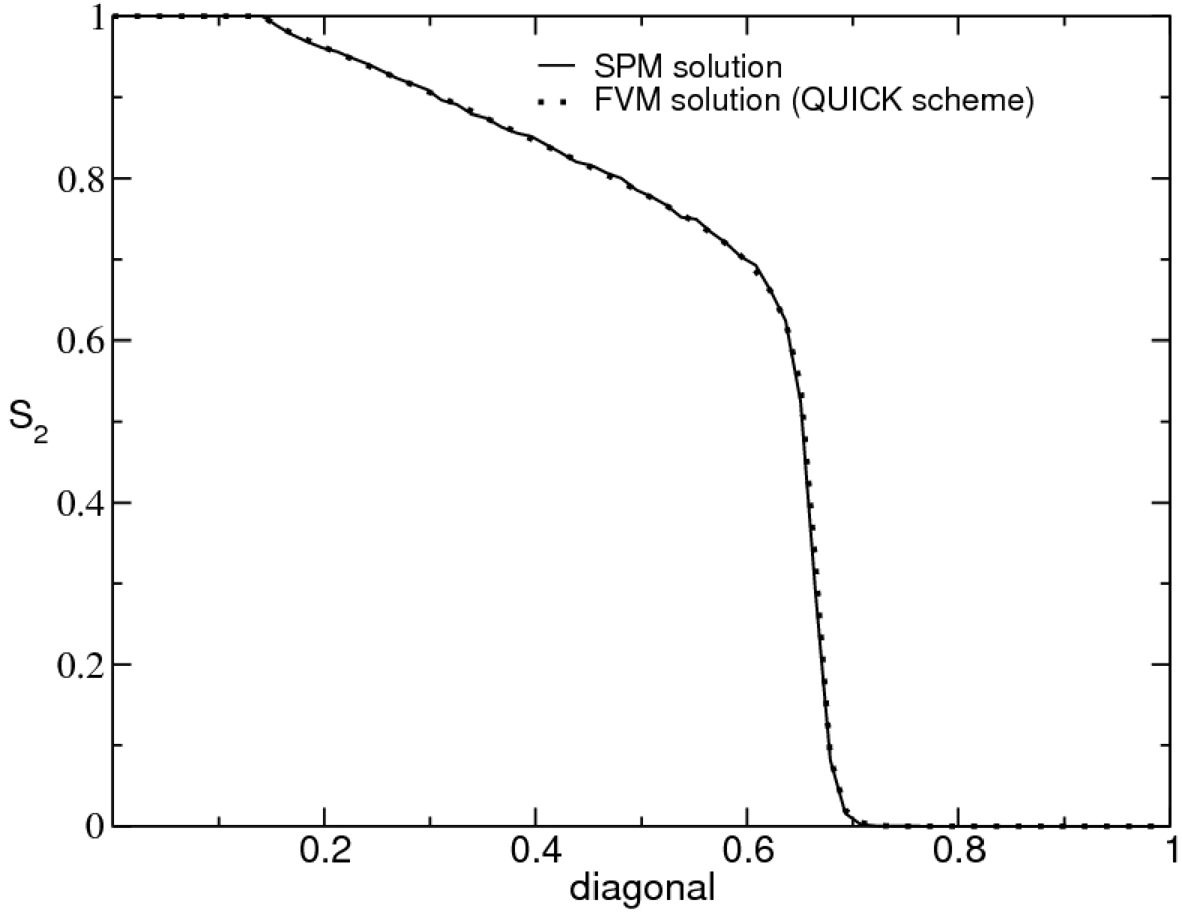
Particle displacement $\dfrac{dx^*}{dt} = u^*$

Small Scale		Large Scale
Particles	← Average Flux	Grid
	Average Mobility →	Poisson equation
		Pressure

2D validation against FVM solution



gas saturation



Stochastic model for particle evolution

Particles perform stochastic motion

$$dx^* = \underbrace{u^* dt}_{\text{drift}} + \underbrace{\sqrt{2D} dW}_{\text{Wiener process}}$$

Particle velocity is given by $u^* = -\lambda^* \nabla p$

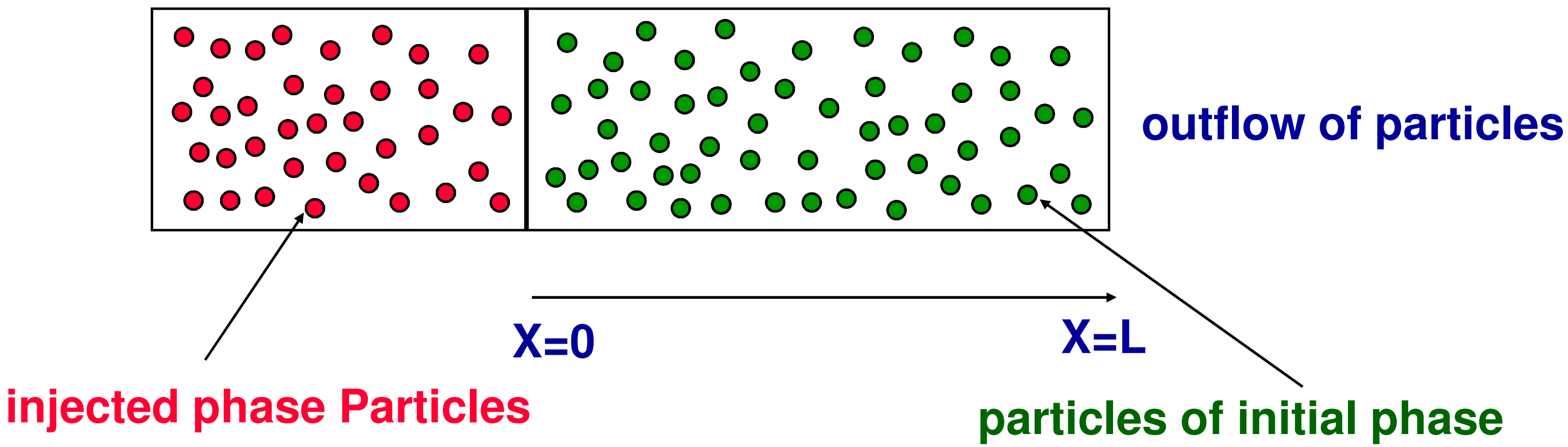
λ^* **= particle mobility**

Langevin model for particle mobility

$$d\lambda^* = -\left(\lambda^* - \lambda_\alpha^{eq}\right)\frac{dt}{\tau} + \sqrt{\frac{2\sigma^2}{\tau}} dW$$

τ **= relaxation time.**
σ^2 **= equilibrium variance**

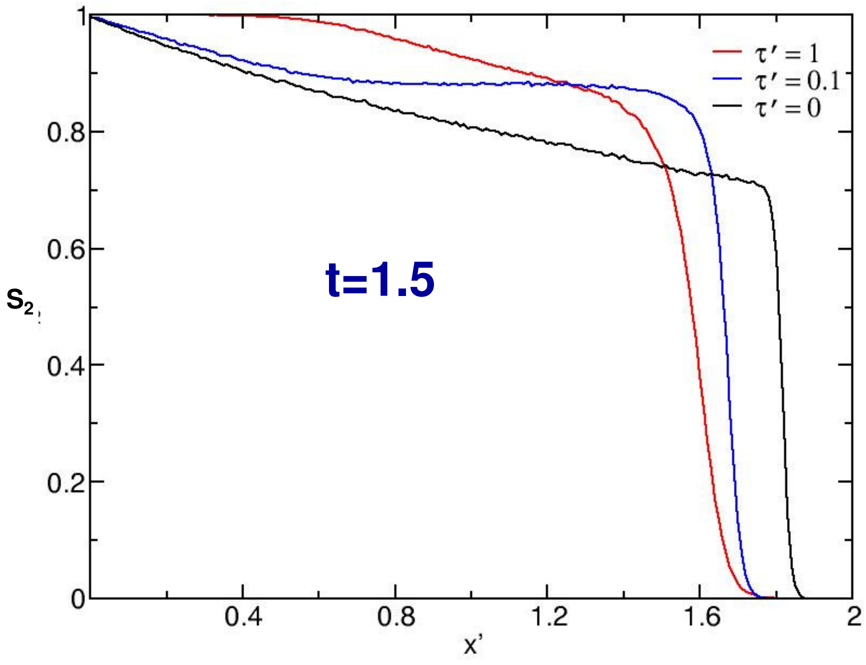
1D Simulation Test Case



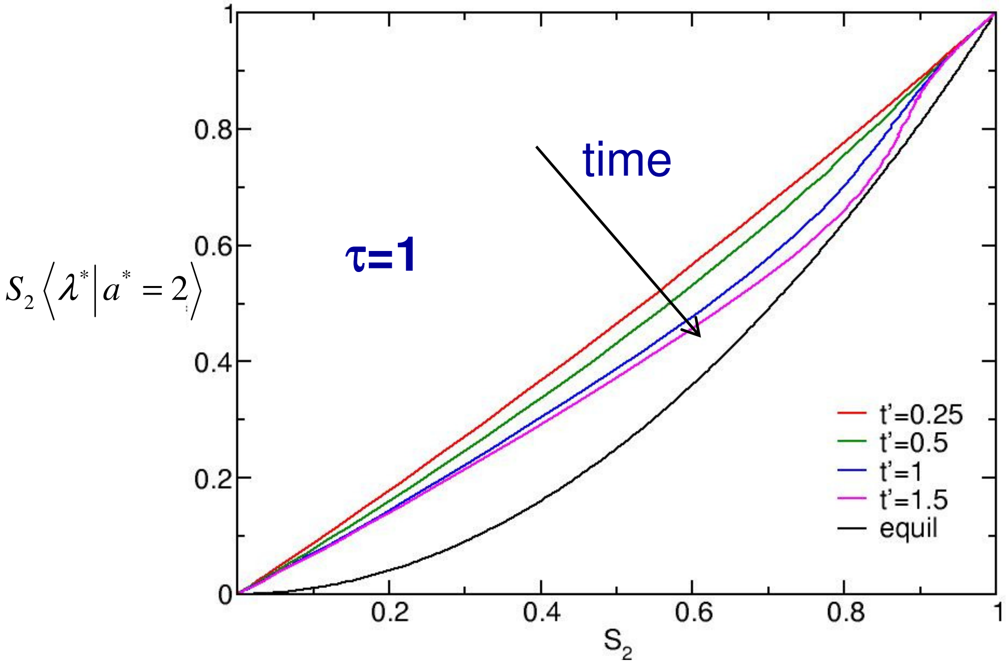
- constant τ, $\sigma^2 = 0$
- Initially all particles are in equilibrium.

Simulation Results

saturation evolution

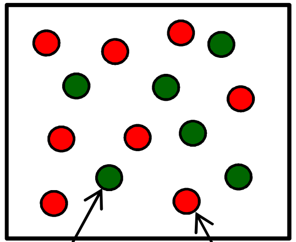


averaged mobility curves



Compositional modeling and joint statistics

Let c^* be the concentration of dissolved CO_2 in brine



$$dc^* = -\frac{1}{\tau_m}\left(c^* - \left\langle c^*\big|_{brine}\right\rangle\right) + \frac{1}{\tau_d}\left(c^* - c^{eq}\right)$$

JPDF $f_{\hat{\lambda},\hat{c},\hat{a}}(\lambda, c, a, x; t)$ **evolves as**

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial x_j}\left\{\left\langle \frac{d\hat{X}_j}{dt}\bigg|_{\lambda,c,a;x,t}\right\rangle f\right\} + \frac{\partial}{\partial \lambda}\left\{\left\langle \frac{d\hat{\lambda}}{dt}\bigg|_{\lambda,c,a;x,t}\right\rangle f\right\} + \frac{\partial}{\partial c}\left\{\left\langle \frac{d\hat{c}}{dt}\bigg|_{\lambda,c,a;x,t}\right\rangle f\right\} = 0$$

Moment equations

$$\frac{\partial \left\langle \hat{c}|\right\rangle S_\alpha}{\partial t} - \frac{\partial}{\partial x_j}\left\{\left\langle \hat{c}\hat{\lambda}|\right\rangle S_\alpha \nabla p\right\} = 0$$

here $\big| = \big|_{a=\alpha}$

$$\frac{\partial \left\langle \hat{\lambda}|\right\rangle S_\alpha}{\partial t} - \frac{\partial}{\partial x_j}\left\{\left\langle \hat{\lambda}^2|\right\rangle S_\alpha \nabla p\right\} + \frac{1}{\tau}\left(\left\langle \hat{\lambda}|\right\rangle - \lambda_\alpha^{eq}\right) = 0$$

$$\frac{\partial \left\langle \hat{c}\hat{\lambda}|\right\rangle S_\alpha}{\partial t} - \frac{\partial}{\partial x_j}\left\{\left\langle \hat{c}\hat{\lambda}^2|\right\rangle S_\alpha \nabla p\right\} + \frac{1}{\tau}\left(\left\langle \hat{c}\hat{\lambda}|\right\rangle - \left\langle \hat{c}|\right\rangle \lambda_\alpha^{eq}\right) - \frac{1}{\tau_d}\left(\left\langle \hat{\lambda}\hat{c}|\right\rangle - \left\langle \hat{\lambda}|\right\rangle c^{eq}\right) + \frac{1}{\tau_m}\left(\left\langle \hat{\lambda}\hat{c}|\right\rangle - \left\langle \hat{\lambda}|\right\rangle\left\langle \hat{c}|\right\rangle\right) = 0$$

Unclosed terms: requires modeling in the deterministic equations

Conclusion and ongoing Work

Conclusions

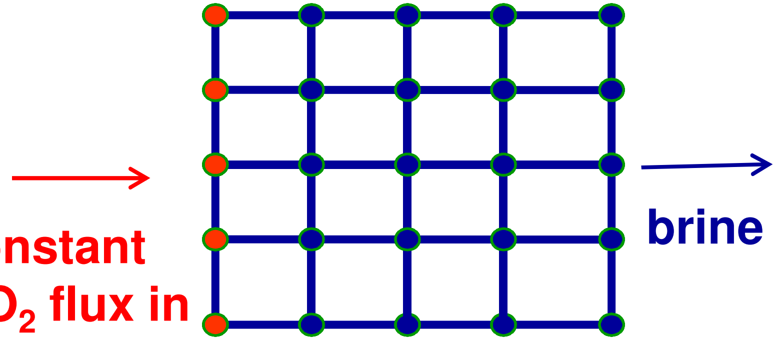
- Lagrangian-stochastic framework for multiphase flow in porous media
- A model for particle position and mobility
- Compositional modeling: importance of joint statistics

Ongoing Work

- Investigation of small scale physics using 2D pore-network model
- Stochastic model for CO_2 dissolution and mixing
- Validation and multidimensional simulations

Thank you

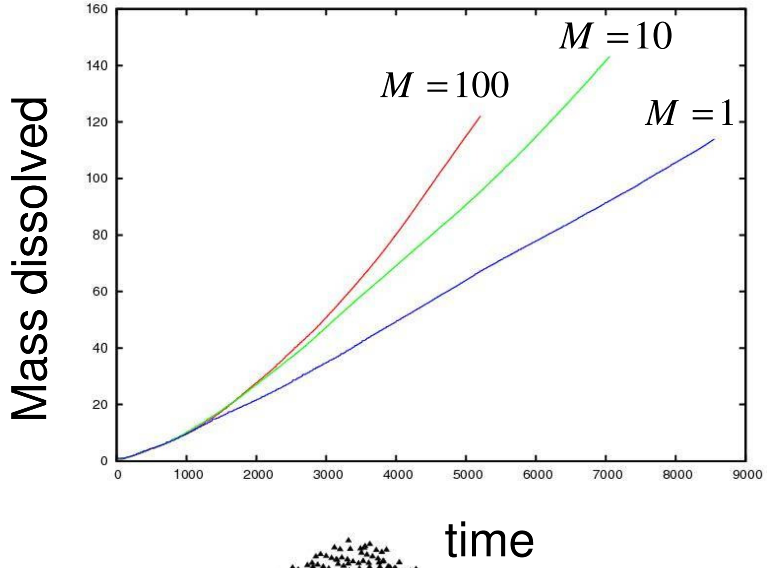
Importance of spatial correlations



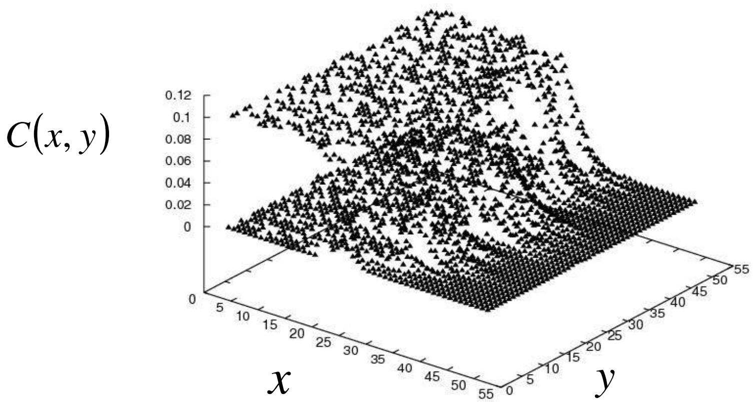
constant CO_2 flux in

brine out

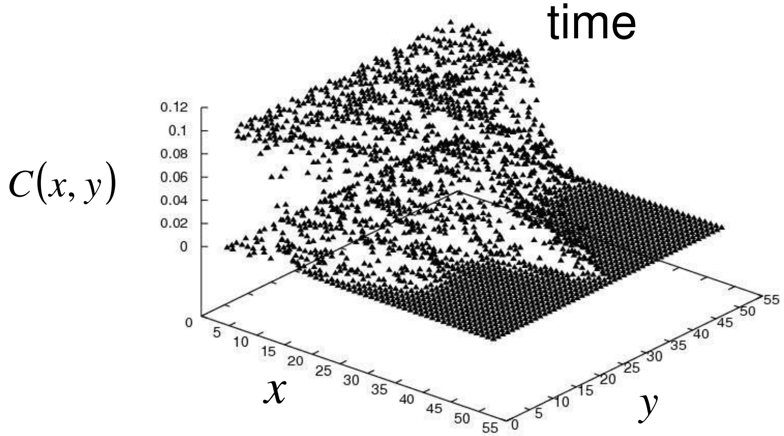
50×50



CO_2 concentration in brine



$M = 1$ stable flow



$M = 100$ unstable flow, viscous fingering