Basic-Math (working title)

Status Quo

Heterogeneous Knowledge Amongst Students

It is a well-established fact that mathematical skills vary vastly amongst students directly after graduating high school (Gymnasium). Nonetheless, a certain level of mathematical proficiency is required for almost any field of study at university. During the study “Basale fachliche Kompetenzen für allgemeine Studierfähigkeit in Mathematik und Erstsprache, Schlussbericht zuhanden der EDK (Eberle et al., 2014)”, it was not only shown that many students lack those skills, but it was also illustrated which mathematical topics and skills are especially important for a successful start at univerity. Those topics and skills have been named “Basale Kompetenzen”, essentially meaning “Basic Competencies [required for starting university]”. However, the strengths and weaknesses of a specific student in terms of those competencies is traditionally not easy to determine. A student would need a detailed map of what the required mathematical skills are, as well as a certain rigor in tracking his own progress, to arrive at an overview of where he stands. Thus, students usually aren’t aware of where their weaknesses lie and whether they’re ready for university. For many students, this means a rough start at university, where many are lost in their lectures after only one or two weeks. Universities can’t provide students with the personalized tutoring and assessment that would be necessary to find their weak points and train them accordingly.

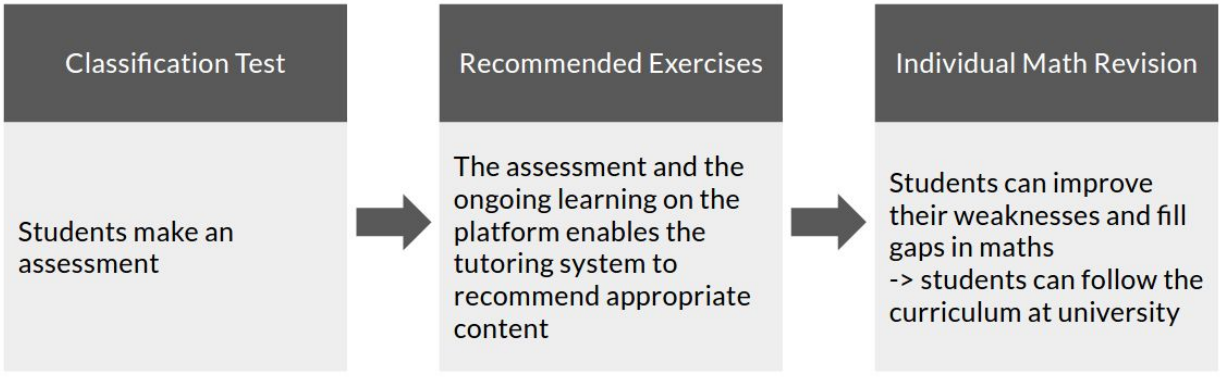
Lacking Data for Research

While students suffer from lack of knowledge, researchers suffer from lack of data. Learning analytics is a complex field with complex data that needs to be carefully generated, collected, and evaluated to get insights. However, often the opportunities and systems to collect said data are hard to come by. When learning is happening offline, the process of collecting and evaluating structured data becomes difficult and expensive. On the other hand, many online systems—for example, Khan Academy—don’t provide Swiss Data Scientists with data, and even if they would, the data couldn’t be generated in respect to the needs of a specific research project. What would be needed is a system in which learners generate a large amount of structured data that is designed along the lines of a specific project.

Solution

Both problems (heterogeneous knowledge and lacking data) could be helped by an ICT solution that guides students along their individual learning paths and collects the generated data. While the students benefit from accurate placement and weakness detection mechanisms,

researchers are provided with valuable structured data that is generated according to their needs. Costs to design such a solution are high, but they can be justified if enough students use the solution and enough valuable data is generated.



Basic Idea Behind the Solution

The relevant topics listed in “Basale fachliche Kompetenzen für allgemeine Studierfähigkeit in Mathematik und Erstsprache, Schlussbericht zuhanden der EDK” (Eberle et al., 2014) should be learnable on an easily accessible online system (web application). Furthermore, the displayed content should be adapted to the field of study in question. Content (Theory and Exercises) for the topics will be provided online. The theory for each topic consists of an explanatory video exclusive to the platform and an explanatory text. Exercises can come in different forms, such as multiple choice or an open solution input. What’s important here is that it’s engaging for the user as well as interesting from a research perspective.

The first interaction with the platform consists of placement test. The idea is to get a good overview of where the student is with his knowledge in the least amount of time possible. Similar to a real private tutor, the system has to ask smart questions in order to get this information out of the student. Furthermore, the questions should be adaptive. Simplified, this means that if the student performs badly on a question, the next one will be easier. The system continues to sharpen its model of the student with each question.

While the algorithms for the placement test also act as a way to train a student’s weakest skills, the student can also select a topic to study himself. When a student selects a topic to study, algorithms provide him with exercises at the right level for this topic. The student can see his progress visualized with statistics and progress bars (see the “wireframes” document for more specific details).

Last but not least, the platform should contain playful elements (gamification) in order to increase the student's engagement. The idea is to give points for each task solved so that the student starts to build toward a high score—a simple yet effective motivational tool.

Artificial Tutor & Assessment

It is crucial for the system to always find and target the weaknesses of the student. As mentioned, the first assessment is a placement test. Afterward, the artificial tutor trains the student in areas he thinks the student needs the most practice with. Analytics help the student to track his activities.

Product

Content Structure

The content is closely related to the insights provided by the report "Basale fachliche Kompetenzen für allgemeine Studierfähigkeit in Mathematik und Erstsprache, Schlussbericht zuhanden der EDK" (Eberle et al, 2014), which investigated which competencies are important for starting at university. The report investigated different skills according to whether they are **a) relevant** and **b) assumed** for different fields of study such as physics, biology, or economics. "Assumed" means that students already have those skills when entering university as opposed to learning those skills at university. For example, calculating with fractions is relevant and assumed in almost every field of study, while statistical methods are often relevant but not assumed. For a first version of the product, we will focus on skills that are relevant **and** assumed in one of the following fields of study provided at EPFL or HSG.[1] Here is a complete list of all mathematical "drill-skills" that were investigated in the report.

Algebra

Alg1	Einfache Berechnungen ohne Taschenrechner durchführen (grosses Einmaleins, Bruchrechnen etc.)
Alg2	Grundoperationen mit Bruchtermen durchführen (Erweitern, Kürzen, Addieren etc.)
Alg3	Mit Doppelbrüchen umgehen (Kürzen, Umstellen etc.)
Alg4	Terme faktorisieren
Alg5	Aufgaben zur direkten und indirekten Proportionalität lösen (Dreisatz und umgekehrter Dreisatz)
Alg6	Potenz- und Logarithmengesetze anwenden

[1] Architecture, Economics, Chemistry and Chemical Engineering, Environmental Sciences and Engineering, Life Sciences and Technology, Communication Systems, Civil Engineering, Electrical and Electronic Engineering, Mechanical Engineering, Computer Science, Mathematics, Microengineering, Physics, Materials Science and Engineering

Alg7	Terme umformen (Äquivalenzumformungen)
Alg8	Lineare Gleichungen lösen
Alg9	Nicht-lineare Gleichungen lösen (quadratische Gleichungen, Wurzelgleichungen, Exponentialgleichungen etc.)
Alg10	Lineare Gleichungssysteme (zwei Gleichungen mit zwei Unbekannten) lösen

Geometie

Geo1	Elementargeometrische Probleme lösen (Fläche des Dreiecks /Kreises, Ähnlichkeit, Satz von Pythagoras etc.)
Geo2	Trigonometrische Berechnungen im rechtwinkligen und allgemeinen Dreieck durchführen
Geo3	Einfache trigonometrische Gleichungen lösen
Geo4	Winkelgrössen vom Gradmass ins Bogenmass umrechnen bzw. vom Bogenmass ins Grad- mass
Geo5	Räumliche Figuren anschaulich skizzieren
Geo6	Grundriss-, Aufriss- und Seitenriss einfacher geometrischer Sachverhalte erstellen
Geo7	Punkte in dreidimensionales Koordinatensystem einzeichnen bzw. Koordinaten von Punkten in dreidimensionalem Koordinatensystem ablesen
Geo8	Geometrische Körper benennen
Geo9	Geometrische Körper skizzieren
Geo10	Formeln für Volumen und Oberfläche von geometrischen Körpern nach einzelnen Variablen auflösen
Geo11	Fehlende Grössen von Körpern berechnen (Kantenlängen, Winkel etc.)
Geo12	Vektoren addieren, subtrahieren, strecken
Geo13	Berechnungen mit dem Skalarprodukt durchführen (inkl. Winkelprobleme)
Geo14	Berechnungen mit dem Vektorprodukt durchführen (inkl. Berechnung von Dreiecksflächen)
Geo15	Berechnungen mit der Geradengleichung im Raum durchführen (inkl. Schnittprobleme)
Geo16	Berechnungen mit der Ebenengleichung im Raum durchführen (inkl. Schnittprobleme)
Geo17	Spurpunkte berechnen

Lineare Algebra

LinAlg1	Vektoren auf ihre lineare (Un-)Abhängigkeit überprüfen
LinAlg2	Linearkombinationen von Vektoren berechnen
LinAlg3	Vektoren nach einer Basis zerlegen
LinAlg4	Lösungsverfahren für lineare Gleichungs- systeme anwenden (z.B. Cramer-Regel, Gauss-Verfahren)
LinAlg5	Rechenoperationen mit Matrizen (Produkt und Inverse) durchführen
LinAlg6	Determinanten von Matrizen berechnen
LinAlg7	Gleichungssysteme mittels inverser Koeffizientenmatrix lösen

Analysis

Ana1	Einzelne Glieder, Summen und Teilsummen von arithmetischen und geometrischen Folgen berechnen
Ana2	Grenzwerte von Folgen und Reihen bestimmen

Ana3	Grundfunktionen (Polynome, Potenz-, Exponential-, Logarithmus-, trigonometrische Fkt.) graphisch in einem Koordinatensystem darstellen
Ana4	Funktionen aufstellen, um Probleme zu lösen („modellieren“)
Ana5	Funktionen spiegeln, verschieben oder zusammensetzen
Ana6	Zusammengesetzte Funktionen in die innere und die äussere Funktion zerlegen
Ana7	Differenzenquotienten von Funktionen berechnen
Ana8	Grundfunktionen mit Hilfe von Differentialquotienten ableiten
Ana9	Tangentengleichungen an Funktionen berechnen
Ana10	Funktionen mit Hilfe der Ableitungsregeln ableiten
Ana11	Flächeninhalt unter Funktionen mit Hilfe von Unter-/ Obersummen berechnen
Ana12	Grundfunktionen mit Hilfe von Integrationsregeln integrieren
Ana13	Extremwertprobleme lösen
Ana14	Kurvendiskussion
Ana15	Volumenberechnungen mit Integralen
Ana16	Lineare, exponentielle und logistische Differentialgleichungen lösen
Ana17	Numerische Verfahren (z.B. Eulerverfahren, Newtonverfahren)

Stochastik

Sto1	Statistische Datensätze grafisch darstellen
Sto2	Lage- und Streumasse sowie Korrelationskoeffizienten berechnen
Sto3	Mit Summenzeichen umgehen
Sto4	Berechnungen im Laplace-Modell (Kombinatorik) anstellen
Sto5	Mehrstufige Zufallsexperimente (als Baum oder Vierfeldertafel) darstellen
Sto6	Pfadregeln für mehrstufige Zufallsexperimente anwenden
Sto7	Technik der Bauminversion anwenden
Sto8	Erwartungswerte berechnen
Sto9	Wahrscheinlichkeit von Intervallen bei der Binomialverteilung berechnen
Sto10	Berechnungen bei der Binomialverteilung durchführen (Quantile, Hypothesentests etc.)
Sto11	Anzahl von Permutationen, Variationen oder Kombinationen bestimmen
Sto12	Terme, die Fakultäten enthalten, umformen und berechnen
Sto13	Terme, die Binomialkoeffizienten enthalten, umformen und berechnen

For different subjects, different skills are relevant. The mapping of which skills are relevant for which topic is found in “Basale fachliche Kompetenzen für allgemeine Studierfähigkeit in Mathematik und Erstsprache, Schlussbericht zuhanden der EDK” (Eberle et al., 2014). The exact list of topics, exercises, and exercise types that will be implemented remains subject to discussion.

In addition, by varying requirements in the proficiency of those “drill-skills”, the surveys have brought attention to a range of other differences between subjects. Those are as follows:

- Reading mathematical representations (Mathematische Darstellungen lesen)

- Producing mathematical representations (Mathematische Darstellungen produzieren)
- Two kinds of exercise types: Problem-solving exercises and Proof-Exercises (Problemlöseaufgaben und Beweisaufgaben)
- Complex Notations (Komplexe Notationen)

For example, in the engineering disciplines, a fair proportion of all exercises are “Proof-Questions”, while in subjects like biology or economics, these are not required at all. Those aspects can be covered in the project by attaching meta information to all exercises on the platform (e.g., {exercise-type: “proof”, complex-notation: “true”}) and providing the different fields of study with exercise types that better match their requirements.

During the content creation phase another focus needs to be on training adaptivity. This means that exercises are crafted in such a way that the different tools for tackling mathematical problems are trained so that the best tool to solve a problem can always be chosen. Another form of important adaptivity is to be able to switch from equations to a graphical display. Last but not least, the vocabulary of the students should become flexible and rich in order to facilitate connections between mathematical topics.

Content Production

Together with Orell Füssli Verlag, DMK, Taskbase and the expertise of University of St. Gallen it is possible to create first class online content. Through previous projects all exponents have experience in creating online content and platforms. The content has to be in German, English and French.

Wireframes

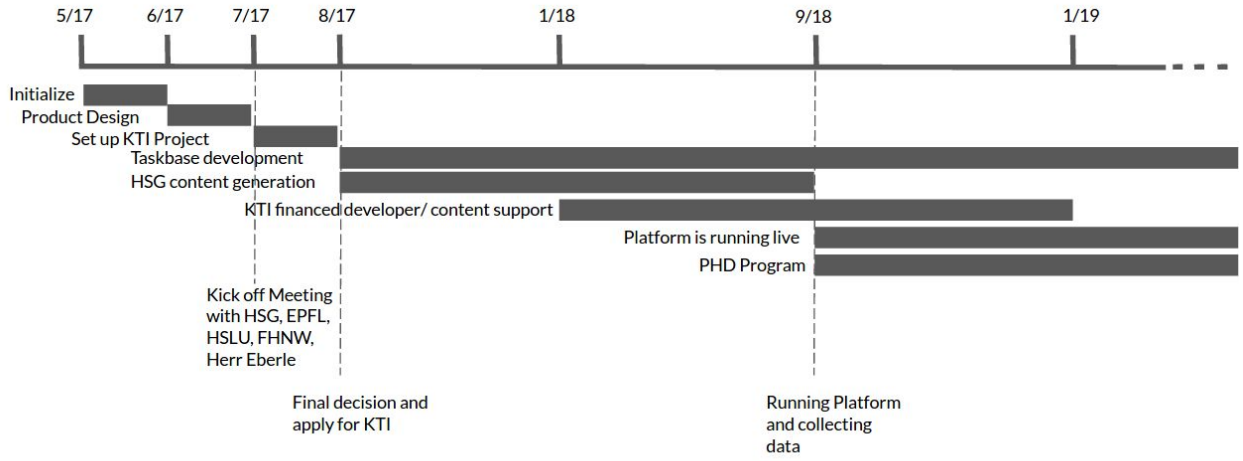
A first draft of what the final product could look like can be found in the wireframes. The basic business logic of the product is described in the following section. A detailed wireframe set can be found here:
https://docs.google.com/presentation/d/1BY4TITEVMyX4z5Eu5xXjB0Uq77oIOAW-5HrB7ajGApw/edit?usp=sharing

When the user first reaches “basic-math.ch” (title may vary), he reaches the so-called “landing page”. This page should promote the value propositions of the product (making the students want to buy a licence) and feature a large “call-to-action” element. For this product, an engaging call-to-action could be “Ready for University?”. When clicked, the student is first asked which subject he is studying, since this is crucial to determining which skills are important to this student. He then completes a placement test that takes from about 30 minutes to 1 hour. The goal of the placement test is to get an understanding of the student’s skills in the least amount of time possible. After taking the placement test (and registering to the platform), the student can choose whether he wants to improve on his weaknesses by using our artificial tutor, which automatically chooses the “right” next exercise, or by specifically targeting a topic himself. For example, if he notices that logarithms are quite important for him at the moment, he can go and

practice just logarithms. For each topic there is also an available theory section featuring a short explanatory video and article. His entire progress is visualized with statistics and progress bars so that the student always has an overview of where he's standing in the process of mastering the required skills to study his subject of choice. The performance of the student has an influence on his overall score, which is used for gamification purposes (see wireframes).

Project Plan

The project is led by three main parties. The first is the University of St. Gallen represented by Prof. Enrico De Giorgi. With his experience in learning systems and his didactical knowledge, he is responsible for the content and the product design. The KTI program finances a developer who works closely with Enrico De Giorgi to onboard content and apply the didactical concepts. The second party is EPFL represented by Pierre Dillenbourg. The KTI program finances a Ph.D. who is responsible for the main research part. The third party is Taskbase. Taskbase is the technology partner and covers half of the costs. A first draft can be seen in the detailed project plan:
https://docs.google.com/spreadsheets/d/1Vf6kz23YgaCoCVBBcdIYMwgspRs_MrAILFPAJJqOO08/edit?usp=sharing



Research

From a research perspective, we have three points of attack. The first goal is to improve our placement tests. This is a crucial task to help students find their weaknesses and recommend exercises and theory tasks to fill their gaps. The second goal is to improve the artificial tutor. The ongoing learning process enables the system to recommend tasks according to personal preferences and moods. This can be exploited. Based on the amount of data, research in this

field is becoming interesting and can lead to new insights. The third goal is to measure the impact of gamification and evaluate different concepts. In the detailed project plan, a first draft of the Ph.D. program can be found. This program contains the biggest part of the research.