Stochastic-Lagrangian Modeling of Multiphase Flow in Porous Media

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April 25, 2008

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Outline

- Motivation
- Multiphase Flow in Porous Media
- Stochastic Framework
- Modeling and Simulation Results
- Conclusion and Outlook
CO₂ Storage in Geological Sites

Complex processes

- Trapping (structural and residual)
- Dissolution of CO₂ into brine
- Reaction of acidic brine with rock
Modeling of flow at different scales

**Small scale model**
- throat (bond)
- pore (site)

**Large scale model**
- $S^\alpha$

**Darcy’s law**
$$ F^\alpha = -\lambda^\alpha (S^\alpha) \nabla p^\alpha $$

**Transport**
$$ \frac{\partial S^\alpha}{\partial t} + \nabla \cdot F^\alpha = q^\alpha $$

**Flow**
$$ \nabla \cdot (\sum \lambda^\alpha \nabla p^\alpha) = -q $$

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...however

- Complex nonlinear small scale processes
- Simple up-scaling of pore scale flow is not enough
- Need of a framework for consistent up-scaling

Stochastic-Lagrangian Framework

pore scale physics

statistics at Darcy scale

Lagrangian statistics

fluid phase 1

fluid phase 2

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Computational Framework

Particles represent phases
Green particles - gas phase
Red particles - liquid phase

Cell averaged saturation

\[ S^\alpha = \frac{\text{number of particles of phase } \alpha}{\text{total number of particles}} \]

Particle displacement

\[ \frac{dx^*}{dt} = u^* \]

Small Scale
Particles

Average Flux

Average Mobility

Large Scale
Grid
Poisson equation
Pressure

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2D validation against FVM solution

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Stochastic model for particle evolution

Particles perform stochastic motion

\[
\frac{dx^*}{dt} = u^* dt + \sqrt{2D} dW
\]

- **drift**
- **Wiener process**

Particle velocity is given by

\[
u^* = -\lambda^* \nabla \rho
\]

\(\lambda^*\) = particle mobility

Langevin model for particle mobility

\[
d\lambda^* = -\left(\lambda^* - \lambda^*_{eq}\right) \frac{dt}{\tau} + \sqrt{\frac{2\sigma^2}{\tau}} dW
\]

\(\tau\) = relaxation time.

\(\sigma^2\) = equilibrium variance
1D Simulation Test Case

- injected phase Particles
- particles of initial phase
- outflow of particles

- constant $\tau$, $\sigma^2 = 0$

- Initially all particles are in equilibrium.
Simulation Results

saturation evolution

\[ S_2(\lambda, a^\tau = 2) \]

\[ \tau = 1 \]

averaged mobility curves

\[ t = 1.5 \]
Compositional modeling and joint statistics

Let $c^*$ be the concentration of dissolved CO$_2$ in brine

$$dc^* = -\frac{1}{\tau_m}(c^* - \langle c^* \rangle_{\text{brine}}) + \frac{1}{\tau_d}(c^* - c^{eq})$$

JPDF $f_{\lambda,\hat{c},\hat{a}}(\lambda, c, a, x, t)$ evolves as

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial x_j} \left\{ \frac{d\hat{X}}{dt}_{\lambda,\hat{c},\lambda,\hat{a},x,t} f \right\} + \frac{\partial}{\partial \lambda} \left\{ \frac{d\hat{\lambda}}{dt}_{\lambda,\hat{c},\lambda,\hat{a},x,t} f \right\} + \frac{\partial}{\partial c} \left\{ \frac{d\hat{c}}{dt}_{\lambda,\hat{c},\lambda,\hat{a},x,t} f \right\} = 0$$

Moment equations

$$\frac{\partial}{\partial t} \langle \hat{c} \rangle S_\alpha - \frac{\partial}{\partial x_j} \left\{ \langle \hat{c}\hat{\lambda} \rangle S_\alpha \nabla p \right\} = 0$$

$$\frac{\partial}{\partial t} \langle \hat{\lambda} \rangle S_\alpha - \frac{\partial}{\partial x_j} \left\{ \langle \hat{\lambda}^2 \rangle S_\alpha \nabla p \right\} + \frac{1}{\tau} \left( \langle \hat{\lambda} \rangle - \lambda^{eq} \right) = 0$$

$$\frac{\partial}{\partial t} \langle \hat{c}\hat{\lambda} \rangle S_\alpha - \frac{\partial}{\partial x_j} \left\{ \langle \hat{c}\hat{\lambda}^2 \rangle S_\alpha \nabla p \right\} + \frac{1}{\tau} \left( \langle \hat{c}\hat{\lambda} \rangle - \langle \hat{c} \rangle \lambda^{eq} \right) - \frac{1}{\tau_d} \left( \langle \hat{\lambda} \rangle - \langle \hat{\lambda} \rangle c^{eq} \right) + \frac{1}{\tau_m} \left( \langle \hat{\lambda} \rangle - \langle \hat{\lambda} \rangle \langle \hat{c} \rangle \right) = 0$$

Unclosed terms: requires modeling in the deterministic equations

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Conclusion and ongoing Work

Conclusions

➢ Lagrangian-stochastic framework for multiphase flow in porous media
➢ A model for particle position and mobility
➢ Compositional modeling: importance of joint statistics

Ongoing Work

➢ Investigation of small scale physics using 2D pore-network model
➢ Stochastic model for CO₂ dissolution and mixing
➢ Validation and multidimensional simulations
Thank you
Importance of spatial correlations

\[ \text{CO}_2 \text{ flux in} \quad \text{brine out} \]

50x50

\[ C(x, y) \]

\[ M = 1 \quad \text{stable flow} \]

\[ M = 100 \quad \text{unstable flow, viscous fingering} \]

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